1953

An evaluation of plastic analysis as applied to structural design, Welding Journal, 32 (5), p. 224-s, (1953), Reprint No. 87 (53-6)

B. G. Johnston

C. H. Yang

L. S. Beedle

Dr. H. A. Neville

Lehigh University

INSTITUTE OF RESEARCH

Welded Continuous Frames & Their Components

Progress Report 8

An Evaluation of Plastic Analysis As Applied to Structural Design

by

Bruce G. Johnston, C. H. Yang, and Lynn S. Beedle

(For Committee Distribution Only)

Welded Continuous Frames and Their Components

Progress Report No. 8

AN EVALUATION OF PLASTIC ANALYSIS AS APPLIED TO STRUCTURAL DESIGN

by

Bruce G. Johnston; C. H. Yang and Lynn S. Beedle

(For Committee Distribution Only)

This work has been carried out as a part of an investigation sponsored jointly by the Welding Research Council and the Department of the Navy with funds furnished by the following:

American Institute of Steel Construction
American Iron and Steel Institute
Column Research Council (Advisory)
Institute of Research, Lehigh University
Office of Naval Research (Contract No. 39303)
Bureau of Ships
Bureau of Yards and Docks

Fritz Engineering Laboratory
Department of Civil Engineering and Mechanics
Lehigh University
Bethlehem, Pennsylvania

18 September 1952
Fritz Laboratory Report No. 205.14

TABLE OF CONTENTS

Progress Report No. 8 :
Welded Continuous Frames and Their Components

AN EVALUATION OF PLASTIC ANALYSIS AS APPLIED TO STRUCTURAL DESIGN

Bruce G. Johnston*
C. H. Yang**
Lynn S. Beedle***

1. Introduction

This article presents a resumé of limitations as well as trends in the application of plastic analysis as applied to structural design. Many of the items discussed are being studied in research projects now in progress at Lehigh University and elsewhere. The results of these investigations may provide at least some of the answers needed to broaden the scope of application of plastic analysis in structural design.

This paper does not purport to be a survey of information on the plastic behavior of structures. Reference will be made primarily to work carried on at the Fritz Engineering Laboratory of Lehigh University as described in progress reports published or pending publication in "The Welding Journal".

In 1946 the Structural Steel Committee of Welding Research Council suggested that work under its sponsorship at Lehigh University should at that time be directed toward the study of fully continuous welded frame construction. Tests of welded seat and top angle connections had been in progress but the opinion was held that the advantages of welding could best be exploited by directing further research toward the fully continuous welded beam

---

* Professor of Structural Engineering, Civil Engineering Department, University of Michigan, formerly Director of Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania
** Former Research Assistant, Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania
*** Assistant to the Director, Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania.

or frame structure. A check on the validity of the assumptions that are usually made in continuous welded frame analysis as applied to conventional or so-called "elastic" design was desired and a complete study of the elastic and plastic behavior of continuous frames and their components was also contemplated so that the current interest in the possibility of utilizing reserve plastic strength in the design of structures might properly be evaluated by actual tests.

The plastic or ductile behavior of steel as used in structural members and frames is important both to elastic and plastic design procedures. In the case of elastic design, the inception of plasticity (on the basis of stress calculations that neglect local and residual stress concentrations) is the essential design criterion, whereas in the case of plastic design the maximum capacity load of the structure is the principal design criterion. Deflection considerations may also be of importance in either elastic or plastic design. At a meeting of the Lehigh Project Subcommittee of the Structural Steel Committee of Welding Research Council on March 24, 1950, the following statement of objectives emphasizing the importance of plastic behavior was approved:

1. To determine the behavior of steel beams, columns, and continuous welded connections with emphasis on plastic behavior, and to develop theories to predict such behavior.
2. To determine how to proportion various types of welded continuous frames to develop the most balanced resistance in the plastic range so that the greatest possible collapse load will be reached.

3. To determine procedures of analysis that will enable one to calculate the collapse loads of welded continuous frames and to verify the analysis by suitable tests.
4. To determine procedures of analysis that will enable one to calculate the elastic and permanent deformations in welded continuous frames in the range intermediate between elastic limit and collapse load.
5. To explore limitations in the application of plastic range design over and above deformation limitations, namely, fatigue, local buckling, lateral buckling, etc.
6. To develop practical design procedures for the utilization of reserve plastic strength in the design of continuous welded frames.

Methods for calculating the ultimate strength of continuous steel beams in the plastic range have long been available. In his "Strength of Materials", Timoshenko (1)* refers to the early work by N. C. Kist who in 1920 proposed a method of determining safe dimensions of steel structures utilizing ultimate load capacity in the plastic range. Referring to this method Timoshenko states "such a procedure appears logical in the case of steel structures submitted to the action of stationary loads, since in such cases a failure owing to the fatigue of metal is excluded and only failure due to the yielding of metals has to be considered". The early tests in Germany by Maier-Liebnitz (2)

---

*Numbers in parentheses refer to references listed at end of article.

seem to have been directed toward removing some of the skepticism regarding the then new ideas of continuous beam and frame design. Settlement of support may cause changes in the stress distribution of such structures in the elastic range but Maier-Leibnitz showed that the ultimate capacity was not affected by such settlements. In so doing he corroborated the procedures previously developed by others for the calculation of maximum load capacity. However, little attempt to actually exploit the use of the ultimate load as a criterion of design appears to have been made at that time. The efforts of Van den Broek (3) in this country and J.F. Baker (4) and his associates in Great Britain to actually utilize the plastic reserve strength as a design criterion are well known and will not be reviewed herein. A review of recent progress in theory of plastic structural analysis has been given by Symonds and Neal (5).

Progress toward the utilization of the plastic reserve strength in steel structures as a design criterion can best be made by a full recognition and study of all of the various factors that affect the behavior of structures above the elastic limit. The study of such limitations, as outlined under items 4 and 5 of the statement of objectives, has been one of the main purposes of the Lehigh investigation.

Before discussing the problems and limitations of plastic analysis as applied to design, it should be pointed out that there is no intrinsic logic in the argument that stress in a steel structure should not go beyond the elastic range. If such an argument is sound, then much of current design practice would have to be abandoned. Specifications for design of both buildings and bridges permit average allowable stresses due to bending, shear, and bearing in the design of pins, rivets, and local points of contact that cause the yield point to be exceeded in local regions of most steel structures. In some cases this is due to neglect of stress concentrations and in other instances of high stress when surrounded by low stressed material. The maximum stresses calculated by simple design formulas are not the true maximum stresses. The latter are not calculated and plastic action is depended upon to insure the safety of the structures since experience has shown that average or nominal stresses form a satisfactory basis for design. Under appropriate conditions, even the average stresses are near the yield stress level. Such localized inelastic behavior usually does not endanger a structure and furthermore most of these same structural members have already experienced greater yield while being straightened in the mill, fabricated in a shop, or forced into position during erection. It actually is during these three operations that the ductility of steel beyond the yield point is called upon to the greatest degree. Having permitted such yielding in the mill, shop, and field, there is no valid basis to prohibit it thereafter, provided such yield has no adverse effect upon the usability of the structure.

Unfortunately, the procedure of basing design on gross approximations for real stress does not always give good results, as is demonstrated by the recent failure of the continuous welded bridge (6) in Canada and other similar failures of structures in this country and abroad. On the other hand, many structures are grossly "over designed" and wasteful of material. Better utilization of the maximum capacity strength of a structure is needed and research in this field will aid in realizing this aim.

In the earliest days of the art the "engineer" intuitively designed structures that, as a result of his experience and feeling for structural behavior, had the required strength and durability that was needed. Although theoretical analysis is now a part of all design procedure, experience is probably still one of the major factors in specification writing. Specifications are primarily the codification of good practice. As the engineer learned analytical methods of elastic stress analysis and coupled these with laboratory test results on the strength of materials and structural members, his attention was more and more drawn to the individual member rather than the whole structure. Now the trend in analysis and in the laboratory is back to a consideration of the complete structure rather than the individual structural component.

Plastic analysis in design, as applied to tier building frames, was foreshadowed by the approximate so-called "elastic" methods of analysis that came into use early in the century for the purpose of calculating stresses due to lateral wind loads. These methods have been reviewed by Clyde T. Morris (7). As early as 1908 methods of analysis were in use based on assumptions that the points of contraflexure in columns were at the mid-height. Such assumptions may lead to a final design result similar to that obtained by an analysis which assumes plastic hinges to develop at the ends of the columns.

2. <u>The Uncertainty of Material Properties in the Inelastic Range</u>

In the elastic range of behavior, up to loads allowed in conventional design, the deflections are very largely determined by the elastic moduli of the material, a constant that varies for steel within rather narrow limits. But in the plastic range, the primary factor determining the structural load-deflection relationship is the lower yield level of structural steel. This is illustrated by the plastic bending behavior of the wide flange structural shape.

Figure 1-a, taken from the First Progress Report (10), shows a small portion of the typical stress-strain diagram for structural steel in the initial plastic range up to the beginning of general strain hardening. Fib. 2-b may serve as an indication of the fact that the plastic bending resistance largely may be realized without using much of the lower yield range that is available. The lower yield level is of obvious importance in plastic analyses and may be defined as that level of stress just sufficient to develop successive new zones of plastic slip in the portions of a test specimen that remains in the elastic state. Fig. 2, taken from the same report (10) illustrates the variation in yield level that may exist in a particular steel at a particular cross section of a wide-flange shape. Keeping in mind the fact that mill tests in general are run at a rate of strain that gives a relatively high yield level estimate, it is of interest to examine a large body of test information furnished through courtesy of the Jackson & Moreland Company. These results are plotted in Fig. 3. More than 3,000 mill tests were analyzed showing a variation of -15% to +25% from the average yield strength level of 39,630 psi. Mill test results are based on the upper yield <u>point</u> rather than on the lower yield <u>level</u> and the latter is more significant in the plastic behavior of structural members. The effective stress strain properties of the material are further modified by the existence of residual stresses caused by cooling, welding, or cold bending.

3. Theories of Initial Inelastic Yield

There is currently much interest in theories and experiments to determine the conditions for experimental "law" for initial yielding of steel or other metals in a state of combined stress. Such laws, however, are not generally required for the prediction of the inelastic behavior of beams, columns, and tension members. Such structural members are governed primarily by uniaxial stress. In other words, the load carrying part of the stress system is one in which two of the three principal stresses are zero or nearly so. The ductile behavior of material in such a uniaxial stress system may be based on the properties determined by the simple tension or compression test.

4. Deflection as a Limitation in Design

One of the principal advantages of plastic analysis is the simplicity by which maximum load capacity may be determined as compared with procedures of indeterminate elastic frame analysis. If deflections must be calculated, much of this advantage is lost, and it must be recognized that deflections do frequently control design, whether it be based on "elastic" or "plastic" analyses. A structure that deflects easily is a flexible structure and motion due to vibration may produce undesirable reactions on the part of occupants or improper functioning of machinery. The upper floors of a few tier buildings are unsuitable for occupancy because of large movements that occur during windstorms. Structural members supporting moving or dynamically applied loads must have certain rigidity to permit proper functioning of moving equipment. Experience with overly flexible suspension bridges is well known. There is a need for more rational determination of proper limits to deflection in various structural applications -- a need that will be intensified in the advent of plastic design procedures.

In elastic design, deflections can be computed with a reasonable degree of certainty in the elastic working load range. In the plastic design procedure such certainty is not always possible, Figure 4, taken from another Lehigh report (13), compares the theoretical and test results for a simply supported 14 WF 30 beam with cantilever sections extending beyond each support loaded so as to simulate a continuous beam between supports. In such case the theoretical curve for load vs. deflection shows three straight line segments. The first bend in the theoretical curve starts when the yield moment is reached at the supports and the second bend occurs when the yield moments are passed at the center of the beam span. Test results in the particular case illustrated fall far short of the theoretically computed values. The initial divergence from the theoretical values in the elastic range is primarily due to residual stress and the lowered strength in the plastic range is due to local and lateral plastic buckling of beam flanges. This result is not necessarily typical but illustrates the divergence that may occur under conditions favoring plastic buckling.

In the case of a beam with heavy flanges and corresponding good resistance to local buckling, with the beam also supported against plastic lateral buckling, the theoretical curve might be exceeded by the test curve.

Load deflection curves shown in Fig. 5 are for two of the portal frames tested under vertical load in another phase of the Lehigh investigations (11). Deflections in each case deviated at low loads from those predicted by elastic theory. Frame No. 1, made up of 8 WF 40 members, however, developed the full plastic strength predicted by simple plastic theory. Frame No. 2, made up of 8 B 13 members, of greater susceptibility to local buckling, did not quite develop its full plastic strength nor did it sustain the plastic strength that was developed. Fig. 6 shows a photograph of Frame No. 1 in its final deflected position.

If many duplicate structures were made to the same design and actually tested to failure, one should expect a considerable scatter due only to variation in yield point (Fig. 3). The factor of safety adopted in plastic design must recognize this uncertainty in maximum load. At allowable loads in plastic design, uncertainties as to deflection must also be tolerated, because of the effect of residual stress. The importance of deflection in plastic design is one of the factors discussed in Progress Report No. 3 of the Lehigh series (8).

In a discussion of deflection it is of interest to refer to the AISC specification for buildings (9), Section 17 of which reads, "the depth of beams and girders in floors shall if practicable be not less than 1/24 of the span, and where subject to shocks or vibrations not less than 1/20. If members of less depth are used, the unit stress in bending shall be decreased in the same ratio as the depth is decreased from that above recommended". The foregoing specification automatically limits deflection. In reference to restrained or continuous spans the next paragraph, Section 17b, reads, "Minimum depth ratios for restrained and continuous spans shall if practicable be such that the deflections at critical points will be not greater than those of simple spans of a minimum depth ratio recommended under paragraph A". A specification of this type must be regarded as one that is based on experience. Evidently, experience has shown that more slender beams are subject to uncomfortable vibration and/or deflections. It is quite probable that in many cases a structure design by plastic procedures might actually meet this specification because of the reduction in deflection produced by continuity even when plastic design is considered. This fact has been discussed in Lehigh Progress Report No. 3 (8).

In summary, the following may be said regarding deflections:

1. In cases where experience, as reflected in existing specifications, has shown a limitation of deflection to be desirable, it will be necessary to consider deflection in the application of plastic design.
2. In cases of plastic design wherein the working loads are considerably greater than those for elastic design, an unavoidable degree of indefiniteness in final deflection at working load must be tolerated.

5. Resistance to Moment in the Plastic Range

In the simple plastic theory it is assumed that "hinge-moments" successively develop at points of maximum moment in structural members. The approximate calculation of maximum load is then based on the strength of the structure incorporating such plastic hinge moments. In view of the obvious importance of plastic bending resistance, the following will outline and discuss in brief some of the factors that may affect the behavior during and subsequent to formation of hinge moments.

5A. Cross-sectional Shape and the Stress Strain Diagram

Annealed structural steel has a nearly linear stress strain diagram up to the yield level after which strains of 10 to 20 times the elastic yield strain occur with no increase in stress.

If a structural steel beam is under pure bending moment, it will start to yield when the fibers furthest from the neutral axis reach the yield point.(Fig. 1) After initial yield the relation between load and deflection of the beam will be non-linear but the beam will resist increasing moment approaching the "hinge-moment" as yielding penetrates toward the neutral axis of the beam. The ratio of the hinge moment to the moment at initial yield is sometimes termed the "shape-factor" of the beam. A wide flange (WF) structural section usually has a shape-factor between 1.10 and 1.20. More compact sections have larger shape-factors; for example, a rectangular beam has a shape-factor of 1.50. Standard I-beam shapes with heavy web and small flanges will have greater shape-factors than wide flange sections with thin webs.

In addition to the effect of cross-sectional shape, there may be secondary effects insofar as shape affects the tendency toward local buckling, lateral buckling, failure by shear, etc., as will be discussed hereinafter. The calculation of the moment-angle curve from a given stress strain diagram for a wide flange section was discussed in Lehigh Progress Report No. 1 (10).

5B. Effect of Shear

If high shear exists in the web of a structural beam over a considerable length, as in the case of a short beam centrally loaded, or a long beam with a concentrated load near the end, the beam may yield rather generally in shear without reaching the usual hinge moments that are assumed in plastic design. This problem of shear as a primary design consideration will be discussed separately as Section 7 of this report. However, in the case of shear combined with large moment, as at a support of a continuous beam, the effect may be somewhat different. The problem is a complicated one since the distribution of shear stress depends on the support details and the way the load is brought into the beam at the support. If the moment fall off rapidly away from the support, initial yielding due to moment may be so localized that strain hardening will commence before any appreciable rotation develops. In such a case, the moment developed at the support may be considerably greater than the hinge-moment predicted by simple plastic theory. In other cases, where the moment gradient is not great, shear may reduce the hinge moment somewhat. The problem of shear in its effect on hinge-moment has been studied by Horne (12) for the case of the rectangular beam section and similar consideration of the I-beam section are now in progress at Cambridge and at Lehigh University.

5C. Effect of Axial Load

The effect of axial load on moment capacity for various combinations of end moment and restraint are reported in Lehigh Progress Report No. 6 (13). In general, the effect of axial load is to reduce the moment capacity of a wide flange structural shape. However, the effect is relatively small in certain cases of small axial load, small l/r ratio, and compact cross-section.

In a building frame, end moments are developed in columns and the combined behavior may be represented by an "interaction" curve in which plotted points represent simultaneously the relative proportion of axial load and bending moment at yield or maximum load. Figure 7, taken from Progress Report 6 (13), shows the interaction curve for a particular case wherein bending moment is applied at one end while the other is held fixed. The upper line indicates the theoretical ultimate strength of a very short column whereas the test results are for an 8 WF column with l/r of 111. It may be seen from the test results that initial yield and column collapse may occur at loads less even than those calculated for initial yield by the secant formula. The test results are not typical of those that may be expected from other combinations of applied load, moment, and slenderness ratio but illustrate what may occur under certain critical conditions coupled with residual stress and local plastic buckling.

In ordinary portal frame columns the ratio of axial to critical load is usually small so that any reduction in hinge moment may be ignored as is done in most studies of collapse strength. However, in the case of tier building structures, the resisting moment of the columns in the lower floors would be reduced by axial load and any evaluation of collapse strength in such a case should include consideration of this effect.

5D. Effect of Local Flange Buckling

As reported in Progress Report No. 1 (10), wide flange structural shapes have a flange thickness sufficient to insure against elastic buckling and will therefore develop the full yield strength of structural steel, except as modified by prior plastic buckling induced by residual stress. If the bending moments are uniformly distributed along a considerable length of beam as in a uniformly loaded beam with moment-free supports, or as in case of concentrated loads equidistant from the center-line, large deflections occur before local deformation results in marked plastic buckling of flanges. However, in tests of continuous beams, the more localized moments over the supports result in considerably greater local rotational deformations than for the simply supported beams. In such a case, for example, local buckling did occur in the relatively thin flange of a 14 WF 30 simulated continuous beam (14). The effect was to lower the maximum resisting moment at the support below the theoretical hinge-moment and to produce a progressive lowering as rotation proceeded. Similarly, in the portal frame connection tests reported in Progress Report No. 4, Part 3 (15) the following is quoted from the conclusions: "While some of the built-up knees have fair rotation capacities most of them collapse very rapidly after first local buckling. This includes those that are well supported laterally. Rotation capacity is dependent on an ability of the knee to resist the tendency to local buckling. Thick flanges and effective lateral support are most helpful". Fig. 8, from Progress Report No. 4, Part 1, illustrates typical local buckling adjacent to a connection. It may be added that thick webs also tend to support flanges against local buckling and improve the situation in this aspect of plastic design.

5E. Effect of Residual Stress

In general, the effect of residual stress in a steel member is twofold: (a) residual stress causes an initiation of yield at loads lower than expected according to usual stress analysis and thus, referring again to Fig. 4, is a major factor in causing the uncertainty of actual deflection at load levels W_{FE}. Since residual stresses are erratic and, although usually present, have variable magnitudes anywhere up to the yield point of the material, the effect on the lower part of the load deflection curve of a structure is obvious. Secondly, residual stresses may also lower the ultimate capacity by inducing either local or general buckling of a compression element or column. This is especially apt to occur if the elements are of intermediate slenderness between the elastic buckling (very slender) range, and the very short and compact member that will develop full plastic yield strength even in the presence of residual stress.

The principal sources of residual stress are:

1. Uneven cooling of rolled structural sections immediately after rolling.
2. Cold straightening, punching, shearing, or bending of sections.
3. Welding.

The effects of cooling and cold bending residual stress have been discussed at some length in Lehigh Progress Report 5 (14). Effects of welding residual stresses have been noted in Progress Report 4 (15) wherein residual stress is considered to have been the cause of local plastic buckling failure in the haunched knees along the inner compression flange. At this location the inner flange is a restrained compression member and it is known from the recent column tests and studies at Lehigh that the residual stress causes early yielding of the outer fibers of rolled flange sections, thus reducing the effective column section. Local buckling first occurs at the point of maximum stress. If this region is localized at the juncture between the haunch and rolled section and if adequate lateral support is provided at this point, the adverse effect of residual stress on the rotation capacity of the joint might be offset.

5F. <u>Effect of Lateral Buckling</u>

In plastic design the concern is primarily with plastic lateral buckling rather than elastic lateral buckling which is guarded against in usual structural design by proper proportions or by lowering of the allowable stress as in the case of compression members. Obviously, in plastic design, proportions of a structure must be such as to eliminate elastic buckling of any type, including elastic lateral buckling. Considering only plastic behavior, local and/or lateral buckling may lower the effective hinge moment in a continuous beam or frame member and may reduce the moment value during continued rotation in the plastic range, thus preventing the realization of the full plastic strength of a frame.

Many tests of frames in the plastic range, often on small models, have involved solid rectangular bar sections bent in the weak plane (3,4). In such a case local and/or lateral buckling is not a problem. Tests by Baker and his associates also include small model frames using wide flange sections but the extrapolation of these results to full size structural members and frames is questionable. Work at Cambridge recently has included tests of nearly full-size continuous frames. The work at Lehigh has put first emphasis on tests of nearly full-scale models of structures, simulating actual practice as closely as possible. Results of these tests have emphasized the need for adequate lateral support if plastic analysis is to be applied to design.

5G. Shape of Cross-section and Longitudinal Distribution of Material

The solution of problems involving bending combined with torsion and/or unsymmetrical bending becomes extremely complex in the plastic range. Furthermore as the plastic range progresses the shape of the cross section may change considerably due to local buckling and/or crippling or crimping such as might occur when a round tube is bent to a sharp angle. In general the contribution of bending to resistance to lateral load will decrease after a certain amount of plastic deformation and the further resistance to lateral load will largely depend on the conditions that obtain at the supports. If the beam is a member of a continuous structure with the supports constrained against lateral movement "catenary" action may develop in which lateral components of the direct tensile force induced by large deflections may offer the primary resistance to lateral load. Such large deflections could not be tolerated as a basis for usual design but might be considered in connection with resistance to bomb blast loads.

All of these factors are complicating influences when plastic analysis of a structure is considered. It is probable that any initial consideration of plastic design primarily should make use of symmetrical sections combined with loading in either principal plane. Plate girders with variable length cover plates probably would not be designed according to plastic theory since the distribution of the material is based on distribution of moments and the elastic and plastic designs therefore would lead to approximately the same weight of structure. Another effect having to do with make up of member is the possible use of built up welded girders made up of rolled plates. These, because of greater depth, may not behave similarly to smaller rolled wide flange sections and the problems of local and lateral buckling will be more acute.

5H. Effect of Encasement

The use of fire proofing or the encasement of steel members and connections for other purposes such as resistance to corrosion may have a considerable effect on the plastic hinge moment and rotation capacity. These effects have been studied by Batho (16) but more research of this type is needed.

6. The Design of Details

It is generally recognized that the most important problems in structural design concern design of details such as connections rather than the design of the main members. Main members are rather thoroughly covered by specifications, but in the design of details the engineer is called upon to exercise the greatest amount of individual judgment and the wisdom that comes only through experience. Here, in the details, the engineer departs the farthest from elastic stress analysis procedures and either by use of approximations based on his own experience and judgment or by use of existing specifications where these cover the case in hand, plastic design in a restricted sense has always been used. A typical example is that of the design for the local compressive stress in a beam web above a support or under a concentration of load. An average stress here of 24 kips per square inch is allowed by the American Institute of Steel Construction Specifications (9). When a beam is supported on a seat angle there is a concentration of stress at the very tip end of the beam so that the yield point is reached at reaction loads which may be lower than the allowable load as calculated by the empirical formulas currently used in design. This fact has been corroborated by unpublished test results based on work done at Lehigh University in 1941 (17). Fig. 9 shows the measured stress distribution for these tests of centrally loaded short 12 WF 50 beams, indicating a reaction of about 22 kips at initial yield. (With loose top angles the reaction at initial yield would be much lower). The allowable end reaction of the beam would have been 45.6 kips. (The AISC Specification seems overly conservative as it would automatically limit the design (reaction load) to 35 kips in this case). Nevertheless, in the test, the only evidence of failure was a gradual spread of the yielded zone from the end of the beam toward the toe of the seat angle. This spread was appreciable at 40 kips but the beam continued to take loads up to 80 kips without serious signs of distress. Figure 10 shows the spread of yield at the 70 kip load and Figure 11 presents curves of reaction vs. deflection at the connection for the various tests that were reported. Above 80 kips the compression yield in the web of the beams had spread to such an extent that barely perceptible local buckling developed as indicated by dial gages but not to an extent easily noted by eye. At maximum load of 90 kips lateral buckling amounted to approximately 1/10 of an inch. At this load the beams had failed rather generally in shear and the tests were stopped. Within the range of the test program the type of seat had no marked effect on ultimate capacity and AISC Specification practice for beam design was shown to be conservative.

Many other examples could be given similar to that of the seat angle connection. In general, where local concentrations of compressive stress exist and where there is a surrounding region of low stressed material or adjacent parts in a built-up member, so as to prevent local buckling, loads many times those causing initial compressive yield may be withstood without serious deformation of the structure as a whole. Furthermore, in these cases where the stress is predominantly compressive, there is little or no danger of failure by fatigue or by brittle fracture.

Other similar cases include the design of rivets and pins in shear, bearing, and (in the case of pins) direct stress due to bending. The high allowable stress in pins for direct stress due to bending (30,000 psi by AISC Specifications) may be justified by the high shape factor for the circular section which is 1.70. Considering plastic behavior there is thus a factor of safety of $1.70 \times \frac{33}{30} = 1.87$ which is greater than the value of 1.65 as sanctioned by AISC for tension members. In the case of the riveted connection, the stresses allowed in bearing and shear would permit local yielding in an individual rivet and adjacent plate, were it not for friction between elements, and, furthermore, in the case of a large connection with many rivets, the end rivets are known to be stressed at much higher levels than those in the interior. The outer rivets must yield and deform first, thus causing a redistribution of load to the various rivets that is entirely analogous to the successive formation of moment hinges assumed in the plastic analysis of the continuous frame.

Since the design of structural details is already based on a restricted application of plastic design theory, it is obvious that in the advent of a broader application of plastic design no great change is to be expected in this very important aspect of structural design. It may be necessary for the engineer to give even more careful consideration to structural details if an overall balanced plastic strength is to be realized. This has been particularly evidenced by the tests of various knees for portal frames as reported in Progress Report 4 (15).

7. Shear as a Primary Design Criterion

Shear as a factor in altering the moment strength of beams has been discussed earlier but shear in itself may be the primary controlling factor in beam or frame design. In their review of progress in the plastic methods of structural analysis, Symonds and Neal (5) state "shear forces can be neglected except in cases where the ratio of span length to beam depth is less than about 4 to 1, which is much smaller than commonly used". The authors have confirmed by correspondence that in using the term "span" they "had been thinking in terms of cantilevers or distances between plastic hinge locations in general", This interpretation is corroborated by the fact that it may be shown that ratios of total span length to depth of 8 to 1 or more may be controlled by shear for currently used wide flange sections under symmetrical loading when fixed at each end. In the case of a concentrated load near the support of a beam restrained at the ends, shear may be the primary design criterion in some cases if the load is at a distance less than four times the depth of the beam from the nearest support. Such load arrangements may occur in the use of offset columns in building frame construction.

Current "elastic" design specifications of AISC permit a lower factor of safety with respect to yield for shear failure in the web of beams as compared with moment failure due to direct stress in the flanges of longer beams. This was brought to the attention of one of the authors as a by-product of the tests on seat angle connections previously referred to (17). In the test set-up a 12 WF 50 beam had a span of 5 ft. face to face of supporting column stubs. As an appendix to the report on web crippling at seat angle supports, the following comment was submitted by one of the authors and is quoted in part and with minor revisions as follows:

> "As a side light on the recent tests of short beams supported by seat angles at each end, certain facts have been noted regarding the

failure of these beams by shear.

"At a total center load of 170 kips or 85 kips shear, each beam had yielded rather generally throughout the beam web area. The average shear stress at failure, based on gross web area, that is, beam depth "d" times thickness "w", gives an average shear stress at failure of 19.1 kips per square inch. There is thus a factor of safety with respect to the allowable average stress of 13 kips per square inch of only 1.47 as compared with the usual safety factor of 1.65 relating the permissable tensile stress of 20 ksi to the minimum specification yield value of 33 ksi. However, the web material had an average yield strength of about 40 kips per square inch, therefore, the factor of safety would only be 1.21 if adjusted to the minimum specification of 33 kips per square inch."

The actual maximum shear stress at failure at the center of the web (neglecting stress concentration in the fillets) was 21 kips per square inch by the formula,

$$v = \frac{VZ}{2Iw}\,^{*} \qquad (1)$$

---

* It is of interest to note that the maximum shear stress is governed by the static moment of the section (Z) which also determines the plastic hinge moment value. If plastic design becomes more prevalent it will be desirable to include in structural handbooks values of Z for WF sections which can be used both for determination of M_p and maximum shear stress. A convenient approximate formula for "Z", the static moment of the complete section, useful in maximum shear stress and plastic moment determination is:

$$Z = (A - A_w)\left(\frac{d - t}{2}\right) + \frac{A_w d}{4} \qquad (2)$$

d = depth of wide flange shape
A = total area
A_w = web area = dw
t = flange thickness

Mr. Walter Weiskopf has pointed out that for those shapes which are split longitudinally to form structural tees, the AISC Manual gives the location of the neutral axis of the half-section. This permits accurate and simple computation of Z for the parent section.

If the allowable value of 13 kips per square inch were for the maximum shear stress rather than the average the factor of safety would be 1.61 for the beam tested with a web material having yield of 40 kips per square inch. This, when adjusted to the mimimum yield value of 33 kips per square inch gives a factor safety of 1.3. Even this is low compared with that usual for direct stress in elastic design. It may be noted that these beams failed at a shear stress level less than that predicted by the shear strain energy criterion or "octahedral shear stress"criterion of failure which predicts yield in pure shear at 19.1 ksi for a material with a yield of 33 ksi. Assuming the optimistic prediction of the octahedral shear stress theory as the criterion of yielding in the web of rolled beams, an allowable maximum shear stress of 11.5 kips per square inch is indicated for a factor safety of 1.65.

The foregoing remarks concerning shear in beams may serve to give added emphasis to the fact that in some cases shear may be expected to be the controlling design criterion in plastic design as well as in conventional "elastic" design.

8. The Limitation of Failure by Fatigue.

The importance of fatigue or repeated load as a modifying limitation on design of beams in the plastic range is apparent to a reader of the summary report on University of Illinois fatigue tests conducted by Professor W.M. Wilson and his associates as reprinted in the Welding Journal (18). There is, nevertheless, some grounds for encouragement on the part of the plasticity design exponent for the case of any structure designed for 100,000 cycles of load or less in which the load goes from a minimum stress of zero to a maximum value. In this range beam sections fabricated with uniform cross-sections and with continuous welds had fatigue strengths in which the initial stresses were above the yield point of the material. On the other hand, built-up beams with partial length cover plates, such as are used in elastic design procedures to distribute the material and utilize it at as high a stress as possible, had lower fatigue strengths than the beams with constant cross-section. On this matter the article concludes, "any plain rolled beam without attachments or flange holes will have a greater fatigue strength than any cover plated beam or any built-up beam of the same or somewhat greater section modulus". Since the application of plastic design is best suited to the use of uniform beam and column sections, the relatively good fatigue strength of such members lends encouragement to the plastic designer.

The uncertain fatigue life of connections such as are used in portal frames (15) presents a limitation to both the "elastic" and "plastic" designer. More fatigue tests of connections such as are commonly used on portal frames and building frames are greatly needed. Some indication of the problem is given by the Fourth Progress Report of the Committee on Fatigue Testing (19). In this report fillet welded T-joints were tested and the stresses for failure at 100,000 repetitions of load were far below the yield point of the material. Simple and economical portal frame connections of the square knee type almost always include a connection of this type at a region of maximum moment. Fatigue failure in such a connection is likely within a few thousand cycles of load but there is a lack of sufficient information to form definite conclusions and until further test work is carried out it seems essential that the application of plastic analysis to structural design be confined, in portal frames, to those cases where only a very few repetitions of maximum loads are to be expected during the life of the structure. An intermediate condition might arise wherein a design for repeated loads at reasonable elastic stress levels might be made with very occasional overloads anticipated due to abnormal wind or snow load conditions. Plastic design then might be used for the consideration of the heavy overloads to be expected only once or twice during the life of the structure.

9. Shakedown

The question of "shakedown" has been explored in detail in research at Brown University. The subject also has been reviewed elsewhere (5). Shakedown is a term applied to a critical load, P_s, intermediate between an upper limit of P_p (maximum plastic load) and a lower limit of P_y (load at initiation of yield), and above which under repeated applications of a certain sequence of load an increment of plastic deformation in the same sense occurs during each cycle of loading, thus leading to ultimate excessive deflections and possible fracture. Below the cricical shakedown load plastic deformation reduces to zero under continued load cycles. On a small scale equivalent to that of the grain structure of the material an explanation of fatigue has been hypothecated in terms not unlike those used in shakedown analysis (20). In some cases the critical shakedown load is equal to the full load P_p and in other cases it may be as low as the elastic limit load P_y. Obviously, under repeated load cycles, shakedown may be an important limitation to plastic design. Shakedown studies necessitate consideration of frame analysis in the elastic range, hence, if required, much of the simplicity inherent in plastic analysis is lost.

10. Strain Aging

Detailed coverage of the problem of strain aging, like that of shakedown, is outside the scope of this paper but it is mentioned as a possible limitation in plastic design. In particular, it may increase the chance of brittle fracture, a limitation to be discussed as the next item. In a general review, Epstein (21) states: "Aging is a change that occurs in the properties of iron or steel at atmospheric temperature or at a moderately elevated temperature after rapid cooling or after cold working...'strain aging' is the term applied to the changes that take place when the final operation consists of cold working. Aging may result in an increase in hardness and strength; a loss in ductility and impact resistance...". Obviously, permission of rather general plastic flow as a design basis in a steel subject to strain-aging would enhance the possibility of brittle fracture. The phenomenon of strain-aging has been noted in tests of ordinary wide flange beams in the plastic range (10).

11. Brittle Fracture

The question of brittle fracture is a most important one since structural engineers have been plagued in recent years with failures of bridges, pressure vessels, and ships wherein the steel has fractured with none of the ductility associated with the usual laboratory tensile test in which considerable elongation, both uniform and local, takes place prior to fracture. The types of fracture occurring in these disasters has more closely resembled that of glass than that normally to be expected in a steel structure. One of the most recent failures of this type is that of the Duplessis Bridge in Canada (6). In the early correspondence that led to the conception of the present article, Mr. F.H. Dill, Welding Engineer of the American Bridge Company, wrote as follows:

> "The rise of the idea that the plastic strength of steel may be utilized to gain greater economy in the design of steel structures is alarming when it is set against the long known and recently reproved fact that structural steel under many common circumstances has NO plastic action. It is alarming even when it is intended to use the reserve plastic strength only to increase the allowable elastic working stresses. Such an increase of allowable working stress will inevitably create more regions of yield point stress and even extend them into primary members. This creates an unacceptable condition because, when there is no plastic action, these stresses can cause fractures and failure of the structure.
>
> "If it could be proved that there is <u>always</u> a predictable amount of plastic action or reserve plastic strength in structural steel, it would be acceptable to count it in the design of structures. Until this condition is proved, however, the possibility that there may be NO plastic action must be recognized

and the designs of structures planned accordingly". Whether or not one agrees with Mr. Dill, his viewpoint is held by a number of structural engineers. Some will argue with good supporting evidence that it is impossible even in "elastic" design to guard completely against the possibility of brittle fracture. Such fractures seem to be caused by an unfortunate coincidence of a number of adverse factors that may in a given structure occur in combination so rarely that the possibility of such brittle failure must be accepted as a calculated risk. Others may argue that a proper combination of good material, skilled workmanship, and adequate structural design of details, can be achieved by specified good practice so as to insure against any possibility of brittle fracture within the range of allowable loads including the possibility of some plastic flow. This is certainly a desirable goal but it is a difficult one to achieve as is attested by the many failures on record. Certainly it seems true that brittle fractures are caused by a combination of adverse circumstances that may include several of the following:

1. Local stress concentrations.
2. Poor welding.
3. Notch sensitive steel.
4. Shock loading.
5. Low temperature.
6. Strain-aging.
7. State of stress combination in which all three principal stresses are tensile.

The problem as applied to bridges has been discussed in general terms by Bijlaard (22) and literally hundreds of references could be cited as examples of individual investigations that have resulted indirectly from failures of welded ships and pressure vessels during and subsequent to the last war. These failures, in spite of improvised measures to reduce them, still occur occasionally and several ships were lost during the winter of 1951-52. However, as has been pointed out, this limitation applies both to elastic and plastic design.

12. Economy of Plastic Design

As a last item for discussion under the general heading of limitations, the question is raised as to whether or not there is any real economy in plastic design procedures assuming that all of the foregoing limitations are adequately answered.

In the use of rolled structural shapes, there is little economy in plastic design if the structure is determinate since, in this case, if bending of wide flange beams is involved, both procedures give essentially the same answer, any difference being due to the shape factor alone. The same is true under certain special cases of continuous beams, as for example, the fixed end beam with a concentrated load at the center. Another instance is that of a three-span continuous beam with the span lengths adjusted in such a way that maximum moments at the center and at the supports are approximately equal.

In connection with the design of structures for high wind loads and maximum snow loads it is to be expected that load magnitudes will occur only a few times in the life of a structure. In such cases limit analysis would seem to have a very definite application especially in design of tier buildings, warehouses, and industrial plants where small permanent deflections could readily be tolerated. A possible exception to the permissibility of such deflections would be in case of industrial mill buildings that have crane runways in which misalignment of runways would create malfunctioning of cranes. Allowable stresses are increased in the elastic design consideration of unusual load combinations that include wind. Plastic design procedures would need the same consideration to effect great saving over the procedures now used. Nevertheless, the utilization of plastic design for these unusual load combinations would be a more realistic approach than the present procedure of simply increasing the allowable stresses. The result would give a structure of balanced strength and there would at least be the possibility of some saving of material. Design studies over a range of variables should be made to actually answer the latter question.

13. Trends in Structural Design

As an intermediate step in the application of plastic design, as applied to tier buildings, the use of "semi-rigid connections" should be mentioned. This has been permitted in principle since 1946 by the AISC "Specifications for the Design, Fabrication and Erection of Structural Steel for Buildings", (9). Semi-rigid connections have been used in the design and construction of building frames in this country and a tentative specification and design procedure is available, (23). As in the case of the current trend in plastic design, the work of Baker and his associates in England was the forerunner of the current use of semi-rigid design in structural steel framing in this country (24).

Referring now to "plastic" design, as permitted in England, British Standard Specification BSS 449 now allows the designer to use the "load factor" design method so long as due account is taken of deformations and accurate methods of analysis are used. Section 29c states, in part:

"..For the purpose of such design accurate methods of structural analysis shall be employed leading to a load factor of 2, based on the calculated or otherwise ascertained failure load of the structure or any of its parts, and due regard shall be paid to the accompanying deformations under working loads, so that deflections and other movements are not in excess of the limits implied in this British Standard".

Apparently, one of the first uses of this specification was in the construction of the gabled continuous welded frames for the new laboratory of the British Welding Research Association at Abington (25). According to the cited reference, the design of this frame showed a reduction of approximately 45 percent compared with "truss and cantilever column" design and 17 percent in comparison with elastic design of a similar welded continuous frame. One reason cited by the authors for the small difference is the fact that greater load factors are required in the specification for plastic design as compared with those covering elastic design. On the other hand, the authors also point out that no lighter frame could have been used because of prohibitive deflections.

In this country, as previously mentioned, the AISC Specification (9) permits for fully continuous beams and girders an increase of 20 percent for stress at the supports as compared to other locations "provided that the section modulus used over supports shall not be less than that required for the maximum positive moments in the same beam or girder, and provided that the compression flange shall be regarded as unsupported from the support to the point of contraflexure". Similarly the specification goes on to permit a combined axial and bending stress in columns of 24,000 psi "when this stress is induced by the gravity loading of fully or partially restrained beams framing into the columns".

It is thus seen that two different approaches to the utilization of plastic reserve strength are currently in process of development. The one, as exemplified by the British Specification, would determine actual "ultimate" or "full" or "limit" loads and divide these by a "load factor", keeping in mind necessary restrictions as to deflection of the structure, possibility of fatigue failure, etc. The other, already partially in use in the AISC Specification, would determine variable permissive stresses due to bending depending on the degree of restraint and distribution of load.

At the moment, possibly the greatest use for methods for analyzing the plastic ultimate strength of steel frames lies in the realm of military applications. Such applications gave impetus to some of the developments of plastic theory in England under the direction of Professor Baker and his group at Cambridge (26). Much work is underway at present on the prediction of strength and behavior during failure of buildings subjected to blast and shock of atomic bomb burst and recent studies indicate the relative superiority of continuous welded frame construction for certain simple types of industrial building frames, (27).

In summary, plastic design methods seem applicable to certain types of construction provided specifications are made available that will give proper attention to the various limitations discussed herein. The following are enumerated as a partial list of examples:

1. Tier building frames with fully continuous welded cons-

truction with some possible exceptions if repeated loads are a possibility.

Plastic analysis should be applicable for design of maximum combinations of gravity load. In the case of lateral wind loads, as has been pointed out, results similar to those obtained by limit design concepts have already been in use for many years. However, the lateral deflections that would occur if plastic moments actually were developed under the lateral loads as applied to tier buildings would be of considerable magnitude and would undoubtedly cause serious cracking of walls and partitions.

2. The design of industrial building frames, wherein plastic analysis procedures might produce a better distribution of material for effective over-all strength against the occasional high wind and/or snow loads to be considered. Special attention would be given to permissive deflection in the case of industrial building frames carrying crane runways which require accurate alignment to insure correct functioning of cranes.

3. Any structure actually designed to absorb dynamic loads such as those resulting from bomb burst or possible collision, such as a ship hitting a dock with too great a velocity, should be designed by plastic analysis procedures. Only thus can the energy existing during impact be absorbed effectively.

14. Summary

This paper has emphasized the limitations inherent in plastic analyses as applied to design. These have included the problems of limiting deflection; reduced plastic moment as affected by shear, axial load, local buckling, residual stress, lateral buckling, etc.; the problem of design of details; shear as a primary design criterion; fatigue; shakedown; and the possibility of brittle fracture. Many of these problems also exist in conventional elastic design. Possible applications for plastic design have been outlined in brief. Some phases of the projects in progress or completed at Lehigh University have been reviewed where pertinent.

By emphasizing limitations the authors do not mean to discredit the possibility of application of plastic analysis to design. A careful study of the problems herein enumerated should stimulate application to those areas of structural engineering wherein plastic procedures do have a place. The goal in structural design is to provide a safe and enduring structure that incorporates maximum possible economy. If plastic analysis can be applied to design to realize these goals, it will be so applied, for the laws of evolution work as surely in the history of man-made structures as they do in the field of biology. Progress in the application of new concepts is slow because the final test is in the actual structure and not in the theory that guides the designer. Theories that do not give full recognition and delineation to limitations in application to practice will deter rather than aid the progress of structural engineering. Hardy Cross in his recent book (28) states:

> "Many articles purporting to be new appear in the field of analysis. Sometimes such articles are useful; often they are harmful. In the field of civil engineering the designers and builders are the men on the firing line".

A sequel to the present article at some future date will present actual design studies and emphasize methods of application to those structures wherein plastic analysis procedures do seem to have a place.

References

(1) Timoshenko, S. "Strength of Materials", Vol. 2, Van Nostrand, 1st. ed. 1930, 2nd ed. 1941.

(2) Maier-Leibnitz "Contribution to the problem of ultimate carrying capacity of simple and continuous beams of structural steel and timber" Die Bautechnik 1:6 (1927).

(3) Van Den Broek, J.A. "Theory of Limit Design", John Wiley & Sons, New York, (1948).

(4) Baker, J.F. "A review of recent investigations into the behaviour of steel frames in the plastic range". J Inst of Civil Engs. 3:185-240, (1949)

(5) Symonds, P.S., Neal, B.G. "Recent progress in the plastic methods of structural analysis" J Franklin Inst 252:383-407, 469-492 (1951)

(6) Merritt, F. S. "Bridge collapse in Quebec charged to brittle steel" Eng News-Record 146:23-4, (1951)

(7) Morris, Clyde T. "Practical design of wind bracing" AISC Publication (1927)

(8) Yang, C. H., Beedle, L. S., Johnston, B. G. "Plastic design and the deformation of structures - Progress Report 3 - Welded Continuous Frames and Their Components" Welding J 30:348s-356s (1951)

(9) American Institute of Steel Construction "Specification for the design, fabrication, and erection of structural steel for buildings" Rev. Feb.1946, p.275, AISC Manual

(10) Luxion, W. W., Johnston, B. G. "Plastic behavior of wide flange beams - Progress Report 1 - Welded Continous Frames and Their Components" Welding J 27:538s-554s (1948).

(11) Ruzek, Jan M.
Knudsen, Knud-E.
Johnston, E. R.
Beedle, Lynn S.
"Welded Portal Frames Tested to Collapse - Progress Report 7 - Welded Continuous Frames and Their Components"
To be published in SESA Proc.

(12) Horne, M. R.
"The plastic theory of bending of steel beams, with particular reference to the effect of shear forces"
Brit Weld Res Assn Rep. (July, 1949).

(13) Ketter, R. L.
Beedle, L. S.
Johnston, B. G.
"Column Strength under combined bending and thrust - Progress Report 6 - Welded Continuous Frames and Their Components"
To be published in Welding J Res Supp.

(14) Yang, C. H.
Beedle, L. S.
Johnston, B. G.
"Residual stress and the yield strength of steel beams" - Progress Report 5 - Welded Continuous Frames and Their Components
Published in Welding J 31:205s-229s (1952)

(15) Beedle, L. S.
Topractsoglou, A. A.
Johnston, B. G.
"Connections for welded continuous portal frames - Progress Report 4 - Part III - Welded Continuous Frames and Their Components"
To be published in Weld J. Res Supp. Nov. 1952

(16) Batho, C.
"The effect of concrete encasement on the behavior of beam and stanchion connections"
Struct Eng 16:427-47 (1938).

(17) Johnston, B. G.
Kubo, G. G.
"Web Crippling at Seat Angle Supports"
Fritz Engineering Laboratory Report 192A2, July 28, 1941.
Published abstract available as AISC Publication No. 168, "Progress Report on Research Projects at Fritz Engineering Laboratory", May 1941.

(18) Anonymous
"Fatigue tests of beams in flexure"
Welding J 30:105s-115s (1951).

(19) Committee on Fatigue Testing — "Fatigue strength of fillet, plug and slot welds in ordinary bridge steel - Report No. 4 - Committee on Fatigue Testing" Welding J 24:378s-400s (1945)

(20) Orowan, E. — "Stress Concentrations in Steel Under Cyclic Load" Welding J 31:273s-282s (1952)

(21) Epstein, S. — "Aging of iron and steel" Metals Handbook (1948) pp,438-443

(22) Bijlaard, P. P. — "Brittle fractures in welded bridges" Eng News-Rec, 146:46-48 (1951)

(23) Hechtman, R. A. Johnston, B. G. — "Riveted semi-rigid beam-to-column building connections" American Institute of Steel Construction Publication No. 206, (1947)

(24) Pippard, A.J.S. Baker, J. F. — "The analysis of engineering structure" Longmans, Green & Co., New York 1936

(25) Atkins, W. S. Lewis, E. M. — "Developments of design and fabrication in recent British structures" Trans Inst of Weld 14:74-84 (1951)

(26) Baker, J.F, Williams, E. L. Lax, D. — "The design of framed buildings against high-explosive bombs" Civil Eng. in War 3:80-112 Published by Inst Civil Eng London (1948)

(27) Johnston, B. G. — "Steel Frames for Industrial Buildings", MIT Conference on "Building in the Atomic Age" (1952)

(28) Cross, H. — "Engineers and Ivory Towers" McGraw-Hill Book Co., 1952

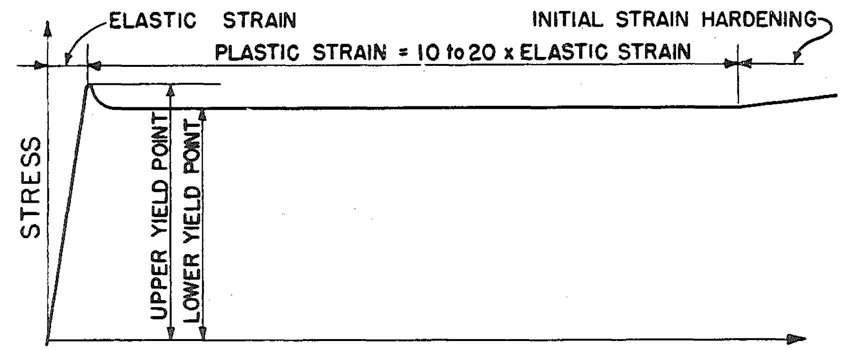


FIG. 1a TYPICAL SHAPE OF STRESS-STRAIN DIAGRAM FOR STRUCTURAL STEEL.

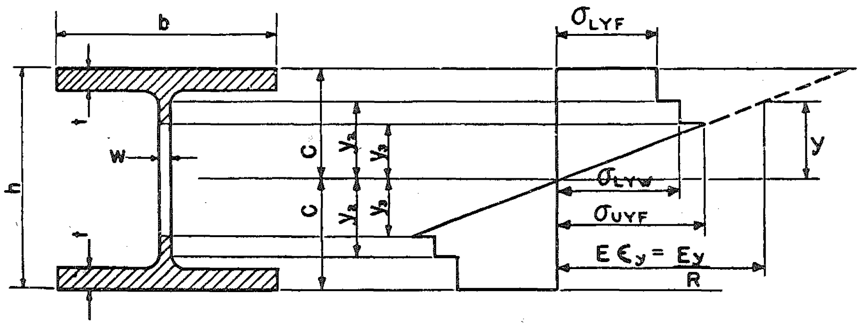
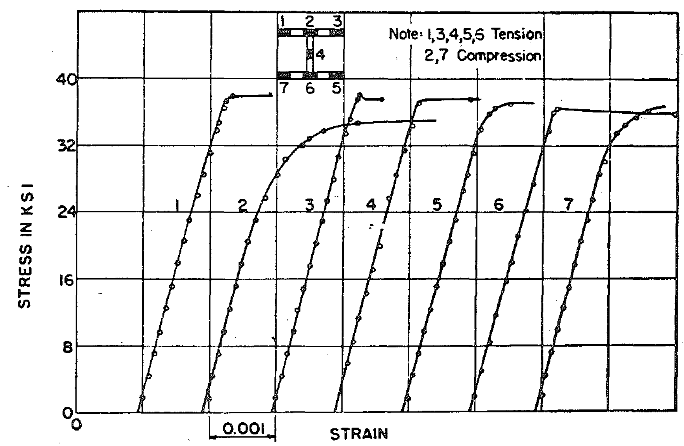
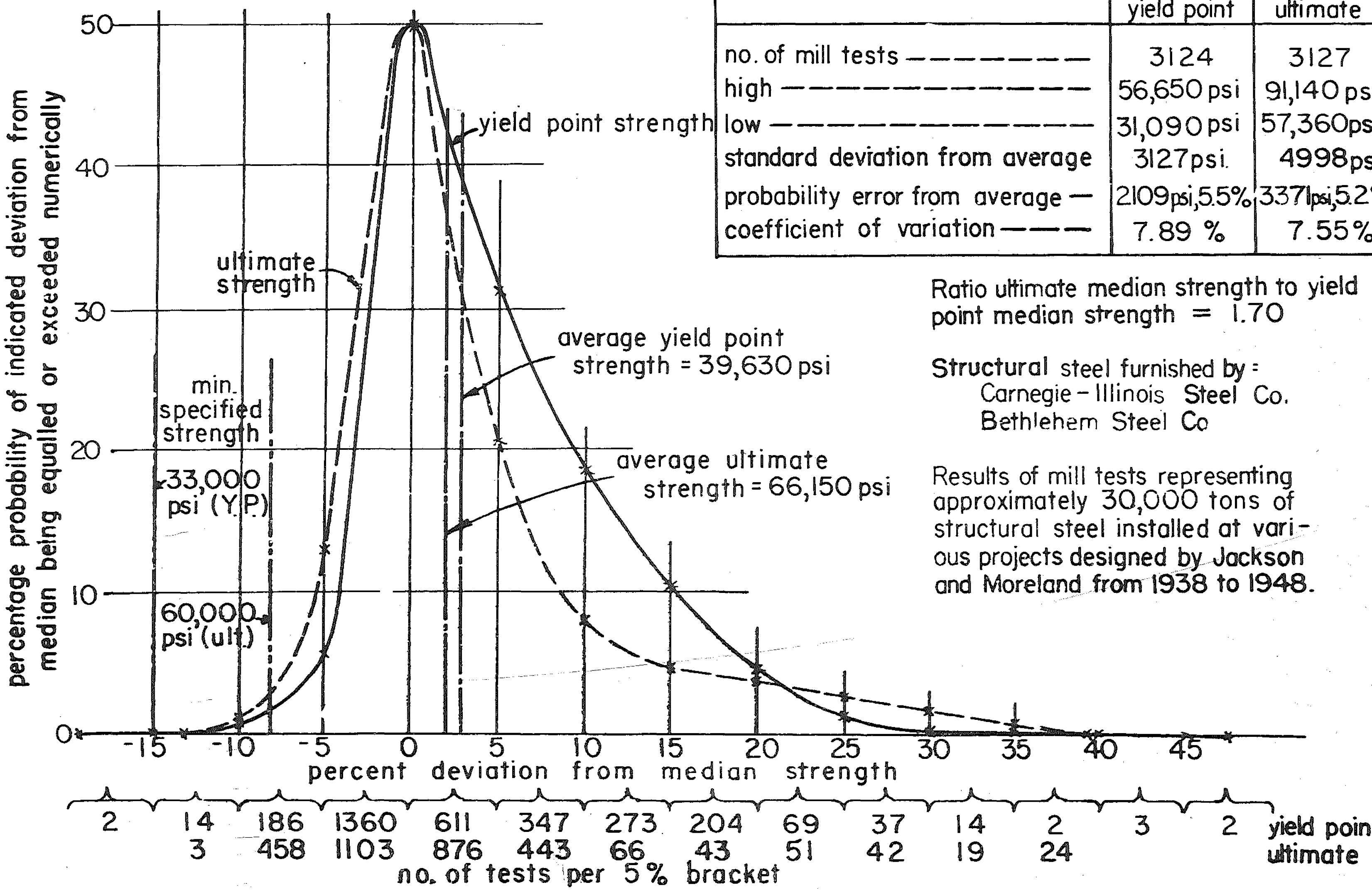


FIG. 1b ASSUMED STRESS DISTRIBUTION IN STEEL BEAM STRAINED BEYOND ELASTIC RANGE.



Stress-strain curves for physical property coupons of 8WF40 as-delivered beam

Fig. 2



	yield point	ultimate
no. of mill tests	3124	3127
high	56,650 psi	91,140 psi
low	31,090 psi	57,360 psi
standard deviation from average	3127 psi.	4998 psi
probability error from average	2109 psi, 5.5%	3371 psi, 5.2%
coefficient of variation	7.89 %	7.55 %

Ratio ultimate median strength to yield point median strength = 1.70

Structural steel furnished by:
Carnegie - Illinois Steel Co.
Bethlehem Steel Co

Results of mill tests representing approximately 30,000 tons of structural steel installed at various projects designed by Jackson and Moreland from 1938 to 1948.

FIG. 3 - RESULTS OF MILL TESTS

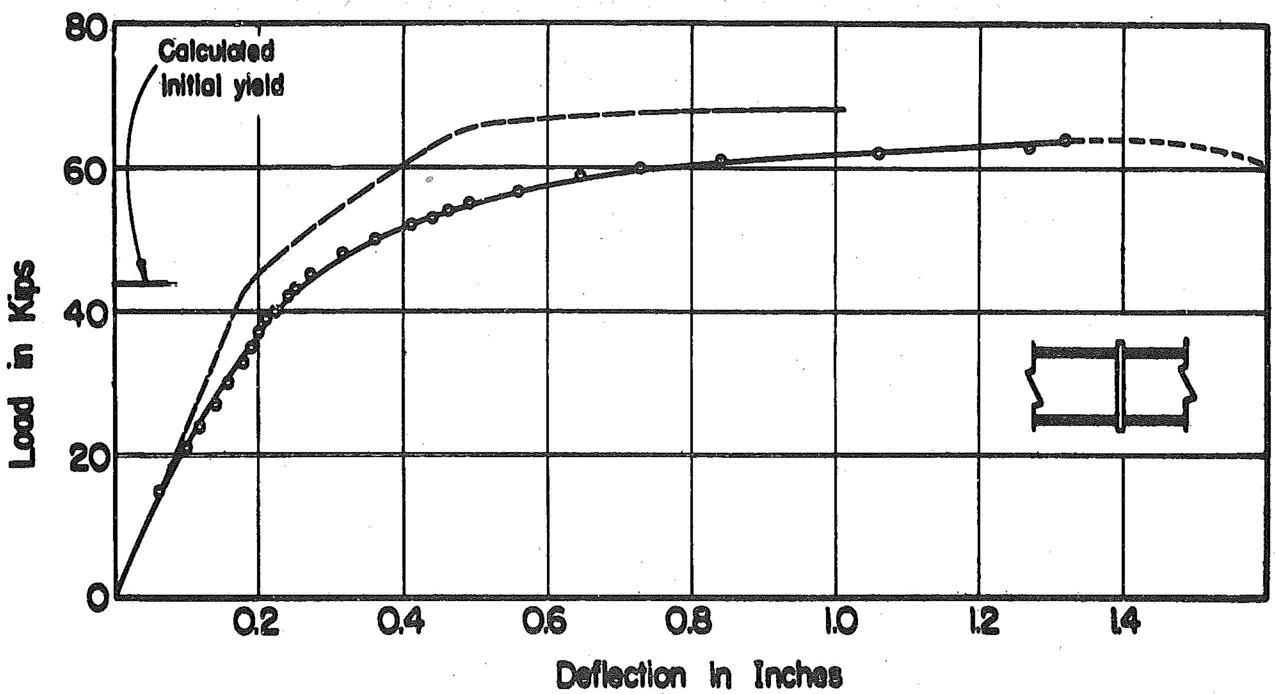


FIG. 4

EXPERIMENTAL AND THEORETICAL LOAD -
CENTERLINE DEFLECTION CURVES FOR 14 WF 30

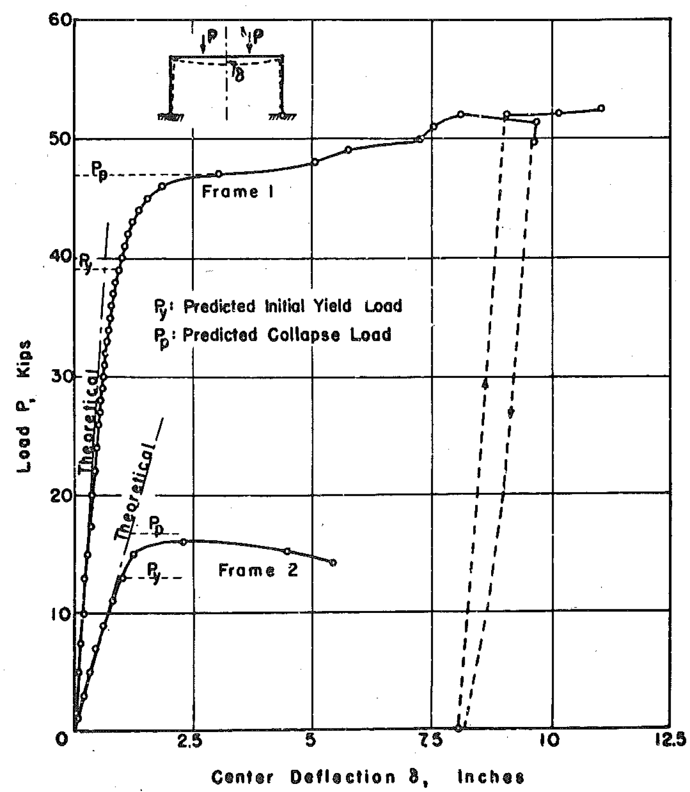


FIG. 5 - DEFLECTION AT MIDSPAN

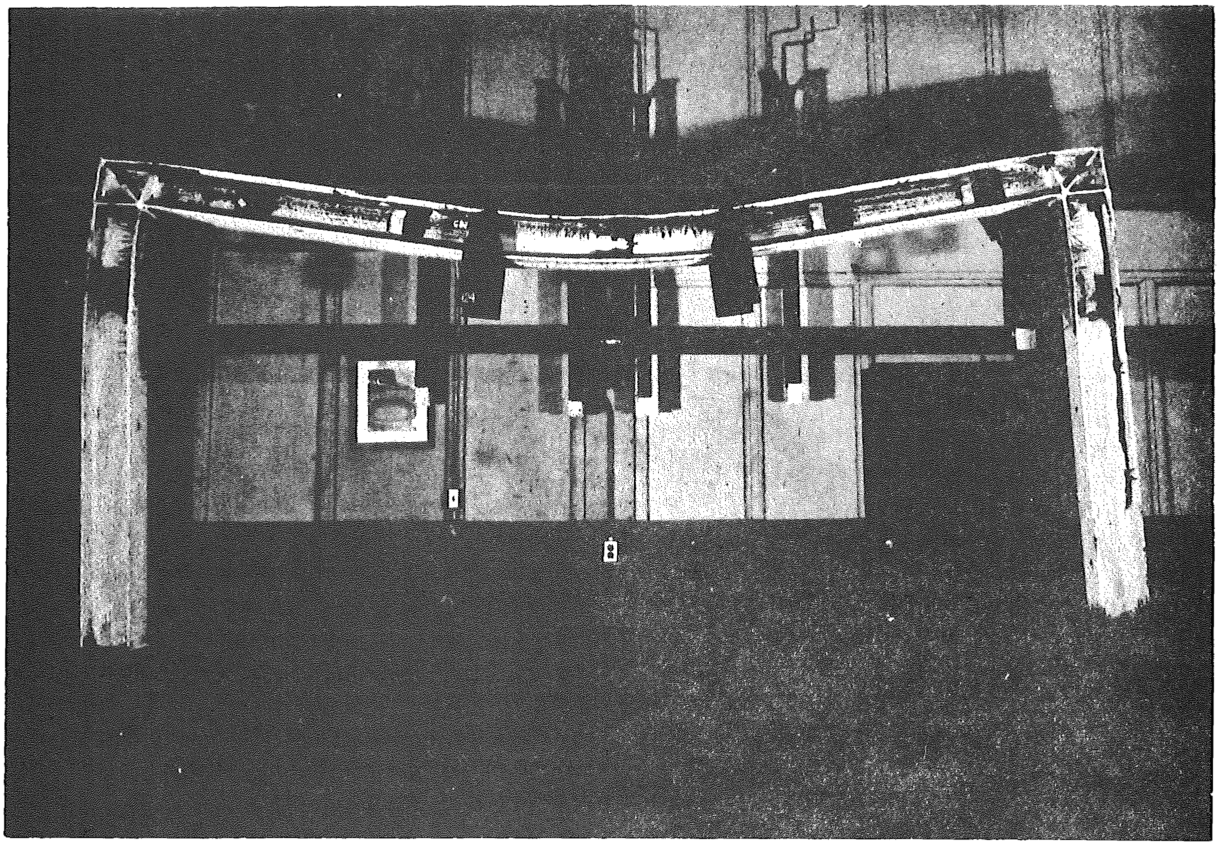


FIG. 6 - FRAME 1 TESTED TO FAILURE

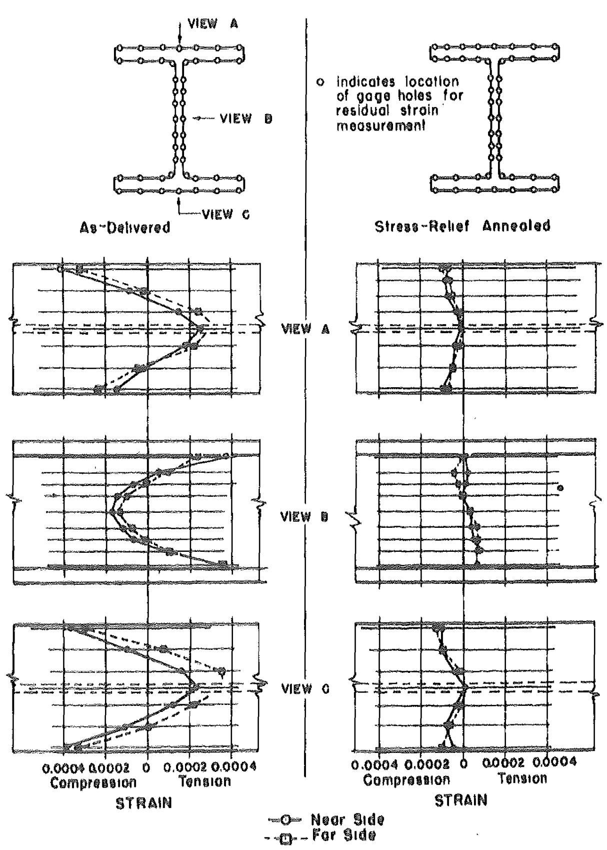
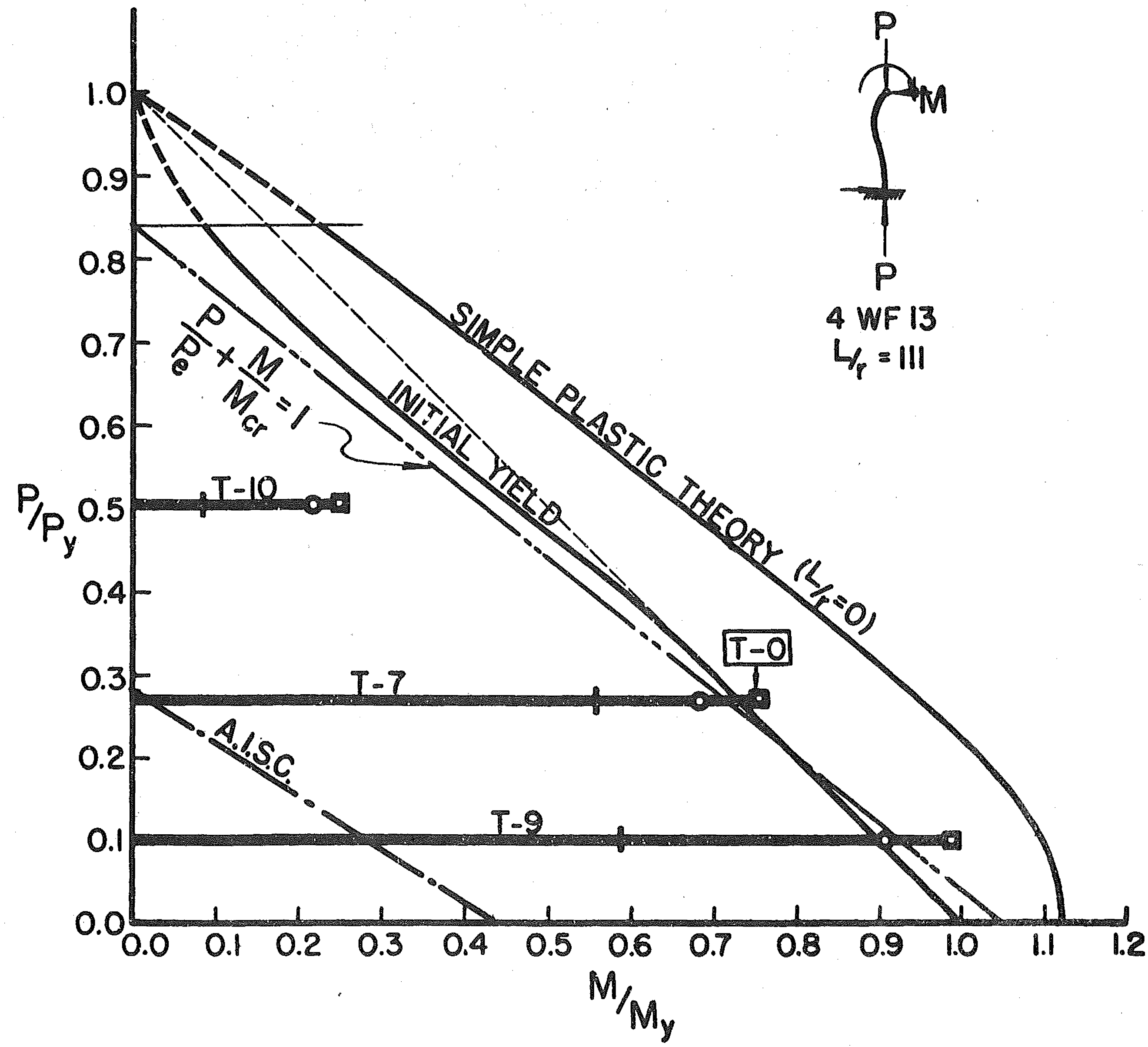
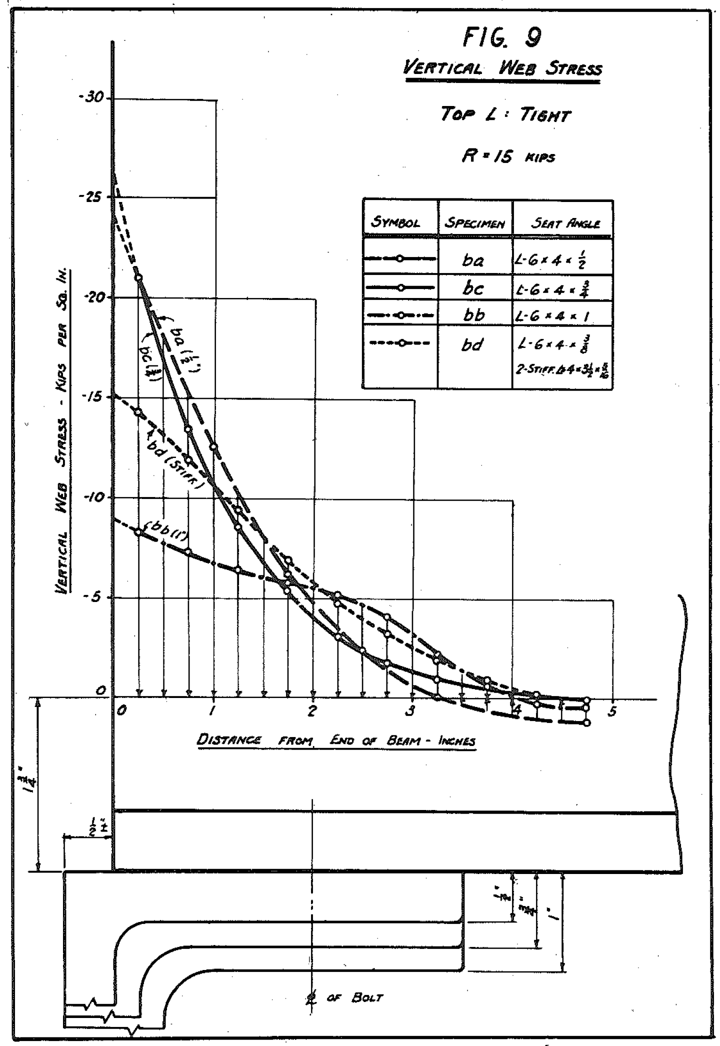


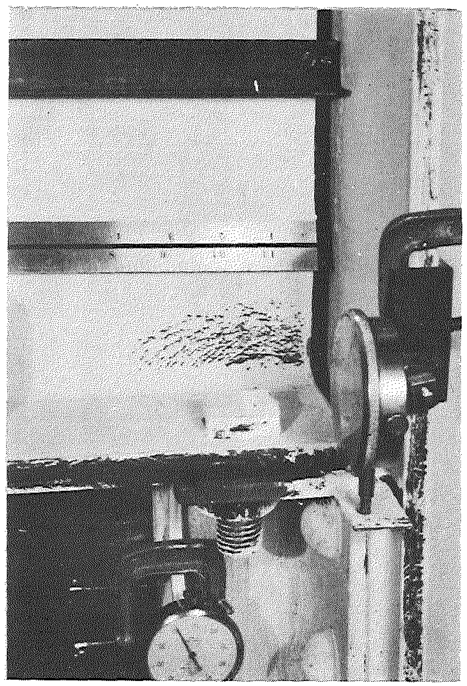
FIG. 8 - RESIDUAL STRESS IN 8WF 67 BEAM



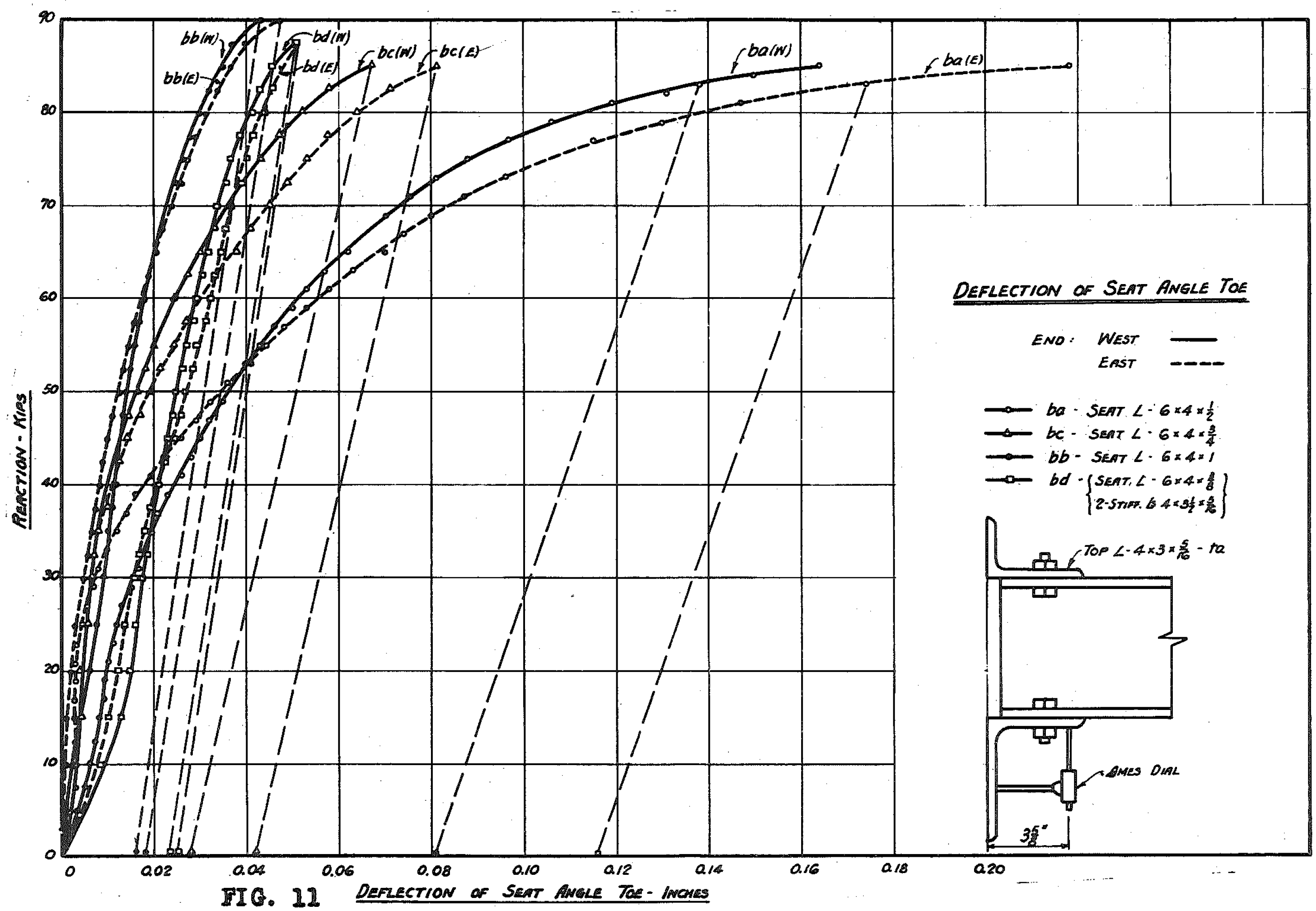
P
M
P
4 WF 13
L/r = 111
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
P/Py
P/Pe + M/Mcr = 1
INITIAL YIELD
SIMPLE PLASTIC THEORY (L/r = 0)
T-10
T-0
T-7
A.I.S.C.
T-9
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
M/My

Fig. 7





70 kips FIG. 10



DEFLECTION OF SEAT ANGLE TOE
END: WEST
EAST
ba - SEAT L - 6×4×1/2
bc - SEAT L - 6×4×3/4
bb - SEAT L - 6×4×1
bd - {SEAT. L - 6×4×3/8 / 2-STIFF. Ls 4×3½×5/16}
TOP L - 4×3×5/16 - 7½
AMES DIAL
3⅝"
REACTION - KIPS
bb(W)
bb(E)
bd(W)
bd(E)
bc(W)
bc(E)
ba(W)
ba(E)
0
10
20
30
40
50
60
70
80
90
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20
DEFLECTION OF SEAT ANGLE TOE - INCHES

FIG. 11